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Hypersonic Turbulent Boundary-Layer Interference Heat Transfer in Vicinity of Protuberances

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Introduction

The presence of a protuberance in a local hypersonic flow results in an interaction between the bow shock wave and the local boundary layer. This interaction results in the formation of a lambda-footed bow shock wave and gives rise to severe aerodynamic heating accompanied by an increase in static pressure in a local region in the vicinity of the protuberance. The present paper utilizes a simple hypersonic flow analysis of the interaction between the oblique portion of the lambda-footed bow shock wave and a turbulent boundary layer on a flat plate.

Turbulent Boundary-Layer Lambda-Shock Interaction

A simplified model that retains the essential features of the interaction between the oblique portion of the lambda shock and the turbulent boundary layer is shown in Fig. 1. It is assumed that the upstream foot of the lambda shock can be treated as a two-dimensional oblique shock, since the analysis is concerned with the interaction at the normal portion of the bow shock.

Experiments described in Ref. 1 revealed that the high pressure downstream of the normal portion of the bow shock feeds upstream in the subsonic part of the boundary layer resulting in a thickening of the boundary layer. This rather abrupt growth of the boundary layer is idealized in Fig. 1 as the formation of an effective "wedge" of angle θ with an "attached" oblique shock such that $p_0 < p < p_{\text{stag}}$.

It is clear that the essential features of the present simplified model for the turbulent boundary-layer lambda-shock interaction are similar to those used in the laminar "weak interaction" problem[2] with the exception, of course, that the turbulent skin-friction law will be used in the present case.

It is well known that the relation between skin-friction coefficient and Reynolds number is nearly independent of Mach number when the fluid properties are evaluated at the wall temperature. Thus, the incompressible form of the skin-friction law expressed as

$$c_{f_w} = C(\text{Rey}_w)^m \tag{1}$$

may be assumed to hold for high speeds, where C and m are constants. Now the skin-friction coefficient c_{f_w} is defined as

$$\begin{aligned} c_{fw} \equiv 2\tau_w/\rho_w V^2 &= (2\tau_w/\rho_0 V_0^2)(V_0/V)^2(\rho_0/\rho_w) \\ &= c_f(V_0/V)^2(\rho_0/\rho_w) \end{aligned} \tag{2}$$

where τ_w is the wall shear stress, c_f is the local compressible skin-friction coefficient based on the freestream dynamic pressure $\rho_0 V_0^2$, and V is the resultant velocity downstream of the oblique portion of the lambda shock corresponding to the pressure p.

Since, for hypersonic speeds, $V/V_0 \approx 1$, then using the equation of state, Eqs. (1) and (2) may be combined to yield

$$c_f = C(p_w/p_0)(T_0/T_w)(\text{Rey}_w)^m \tag{3}$$

Assuming an n-power law for the viscosity-temperature relation and $V/V_0 \approx 1$, it follows that the wall Reynolds number at station B, Rey_w, and the freestream Reynolds number, Rey_0, are related as

$$\text{Rey}_w = (p_w/p_0)(T_0/T_w)^{1+n}\,\text{Rey}_0 \tag{4}$$

Substituting Eq. (4) into Eq. (3) and recognizing that $\partial p/\partial y = 0$ in the boundary layer gives

$$c_f/(\text{Rey}_0)^m = C(p/p_0)^{1+m}(T_0/T_w)^{1+m(1+n)} \tag{5}$$

For hypersonic speeds, the temperature-Mach number relation, assuming a recovery factor of unity, may be approximated as

$$T_w/T_0 = 1 + [(\gamma - 1)/2]M_0^2 \approx [(\gamma - 1)/2]M_0^2 = \xi_0 \tag{6}$$

so that Eq. (5) becomes

$$c_f/(\text{Rey}_0)^m = [C/(\xi_0)^{1+m\,(1+n)}](p/p_0)^{1+m} \tag{7}$$

This is the expression for local skin-friction coefficient c_f at station B that takes into account the interaction between the oblique portion of the lambda shock and the turbulent boundary layer.

In the absence of the protuberance, i.e., the *noninteraction case*, $p/p_0 = 1$, and Eq. (7) reduces to

$$c_{f_0}/(\text{Rey}_0)^m = C/(\xi_0)^{1+m\,(1+n)} \tag{8}$$

where c_{f_0} is referred to as the turbulent skin-friction coefficient at station B without interaction. It follows, therefore, that the effect of the turbulent boundary-layer lambda-shock

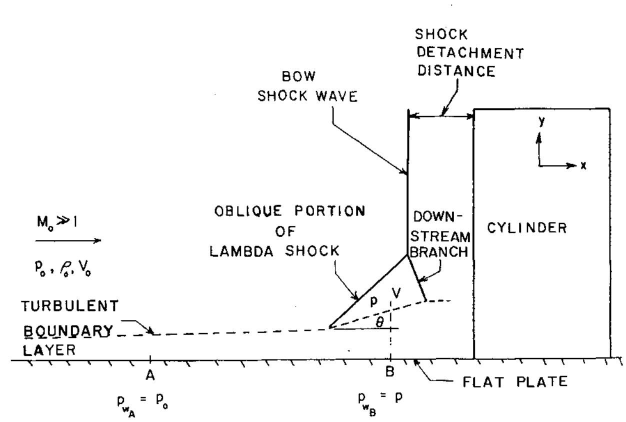

Fig. 1 Interaction of lambda shock and turbulent boundary layer (schematic).

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interaction is simply

$$c_f/c_{f_0} = (p/p_0)^{1+m} \tag{9}$$

It now remains to determine the pressure ratio p/p_0 across the oblique portion of the lambda shock as a function of a turbulent boundary-layer lambda-shock interaction parameter.

Interaction Pressure Rise

An approximate expression for the pressure ratio as a function of the freestream Mach number M_0 and the "wedge" angle θ, may be shown to be[2]

$$p/p_0 = 1 + \gamma M_0 \theta \tag{10}$$

where $M_0\theta$ is the hypersonic similarity parameter, and θ is related to the boundary-layer displacement thickness δ^* as

$$\theta = d\delta^*/dx \tag{11}$$

Now, again basing the fluid properties on the wall temperature, δ^* can be approximated by the incompressible form

$$\delta^* = C_1 x (\text{Rey}_w)^\omega \tag{12}$$

where C_1 and ω are constants. Thus

$$\theta = C_2 (\text{Rey}_w)^\omega \tag{13}$$

where $C_2 = (1 + \omega)C_1$.

Substituting Eq. (13) into Eq. (10) and using Eqs. (4) and (6), the pressure ratio may be written, to first order, as

$$p/p_0 = 1 + \text{K}(\text{Rey}_0)^\omega (M_0)^{1-2\omega(1+n)} \tag{14}$$

where $K = \gamma C_1(1 + \omega) [2/(\gamma - 1)]^{\omega(1+n)}$

Likewise from Eqs. (9) and (14), to first-order terms,

$$c_f/c_{f_0} = 1 + K_2(\text{Rey}_0)^\omega (M_0)^{1-2\omega(1+n)} \tag{15}$$

where $K_2 = (1 + m)K$. It now remains to use Eqs. (14) and (15) to find correlation formulas for maximum interference heat transfer and corresponding pressure rise as a function of a turbulent boundary-layer shock-interaction parameter, χ_t, which is identified as

$$\chi_t = (\text{Rey}_0)^\omega (M_0)^{1 - 2\omega(1+n)} \tag{16}$$

The results thus obtained should be considered a first approximation, which yields interaction parameters that are useful for the correlation of experimental data.

Correlation Formulas for Maximum Interference Heat Transfer and Pressure Rise

Using the Reynolds analogy between skin friction and heat transfer, Eq. (15) may be written in terms of the film coefficient of heat transfer as

$$h/h_0 = 1 + K_2 \chi_t \tag{17}$$

where the explicit form of the interaction parameter χ_t is determined by choosing appropriate values of ω and n. For $\omega = -, \frac{1}{5}$, as in Ref. 3, and $n = 1$,

$$\chi_t = (M_0)^{9/5}/(\text{Rey}_0)^{1/5} \tag{18}$$

Typical experimental values of $(h/h_0)_{\max}$ from Ref. 1 have been plotted in Fig. 2, using Eq. 18 in a correlation formula similar in form to Eq. 17. It is seen that reasonable correlation is obtained even though the data are for Mach numbers somewhat below the hypersonic range.

It follows from Eq. 14 that the correlation formula for pressure rise, $\Delta p = p - p_0$, should be of the form

$$\Delta p_0/q_0 = K/(M_0 \text{Rey}_0)^{1/5} \tag{19}$$

where $q_0 = (\gamma/2)p_0 M_0^2$ is the dynamic pressure of the freestream.

In Fig. 3, the experimental data of Ref. 4 have been plotted using the correlation formula, Eq. (19). Although the experimental data are for Mach numbers below the hypersonic range, it appears that the effect of both Mach number and Reynolds number on the pressure rise is correlated.

For the interaction of the lambda-footed shock and a *laminar* boundary layer, one may use $m = -\frac{1}{2}$, $n = 1$, and $\omega = -\frac{1}{2}$. This yields correlation formulas for skin friction and heat transfer as a function of the well-known laminar boundary-layer shock-interaction parameter, $\chi_{\text{lam}} = M_0^3/(\text{Rey}_0)^{1/2}$.

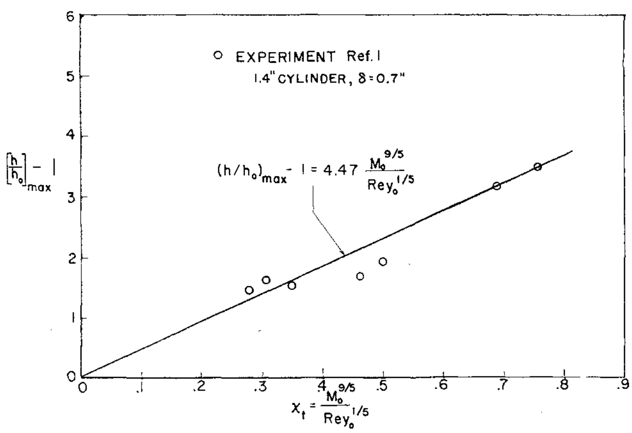

Fig. 2 Correlation of experimental data for maximum interference heat transfer for cylindrical protuberance-turbulent boundary layer.

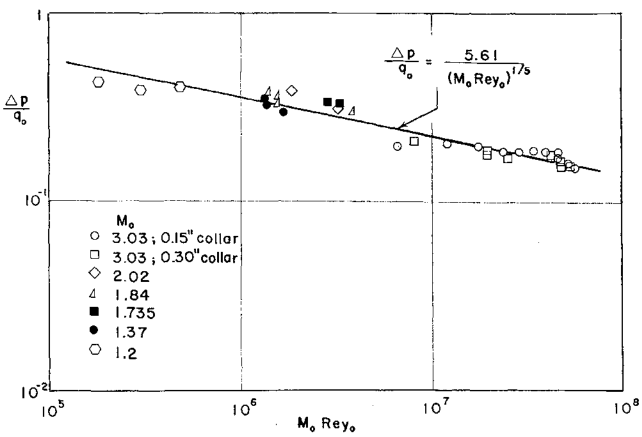

Fig. 3 Correlation of experimental data for interference pressure rise for turbulent boundary layer.

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